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Flexible Prescribed Performance Output Feedback Control for Nonlinear Systems with Input Saturation

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***Abstract*—A flexible prescribed performance control (FPPC) approach for input saturated nonlinear systems (ISNSs) with unmeasurable states is first presented in this article. Compared to the standard prescribed performance control (SPPC) or funnel control methods for ISNSs, the "flexibility" of the proposed FPPC algorithm is reflected in two aspects: (1) the proposed FPPC algorithm simultaneously considers multiple key indicators (including the steady state accuracy, convergence time and overshoot), which are widely demanded in industrial production; (2) the proposed FPPC algorithm achieves a trade-off between performance constraint and input saturation, i.e., the performance boundary can adaptively increase when the control input exceeds the saturation threshold, effectively avoiding singularity; conversely, when the control input is within the saturation threshold range, the performance constraint boundary can adaptively revert back to the original performance boundary. In addition, the unmeasured states are observed by the state observer, and the unknown nonlinear functions are approximated by fuzzy logic systems (FLSs). The results demonstrate that the proposed output feedback control algorithm can ensure that all closed-loop signals are semi-globally bounded, the system output can track the desired signal within a prescribed time, and the tracking error is consistently maintained within flexible performance boundaries that depend on input and output constraints. The developed algorithm is exemplified through simulation instances.**

***Index Terms*—Flexible prescribed performance control, input saturation, output feedback control, fuzzy logic systems.**

I. Introduction

DUE to physical limitations, production safety, etc, the physical system is inevitably subject to constraints. These constraints can generally be categorized into two types: those related to control performance, including steady-state accuracy and convergence time; and those related to system carrying capacity, including state constraints and input saturation. The violation of these constraints may lead to a degradation in system performance and pose potential threats to system security. The adaptive control of constrained nonlinear systems has thus garnered considerable research attention, and many effective control methods, like the barrier Lyapunov function (BLF)-based approaches [1]–[7], the nonlinear mapping function (NMF)-based approaches [8]–[13], and so on. In order to satisfy certain transient performance and steady-state performance of the system, the PPC pioneered in [14] was developed to deal with the performance constraints by introducing a prescribed performance function (PPF). Then, the PPC method was extended to nonlinear systems with different forms [15]–[19]. Note that the above PPC (Hereinafter referred to as "SPPC") approaches only guarantee that the tracking error satisfies the predefined tracking accuracy in infinite time, which limits the application scope of the method to some extent. In practice, users often anticipate the tracking error to converge to a pre-set tracking accuracy within a finite time.

The achievement of system stability within a finite time is widely acknowledged to hold significant importance. In the past few decades, numerous finite/fixed-time control approaches for nonlinear systems were presented [10]–[13], [20]–[27]. The finite/fixed-time control methods presented in [10]–[13], [20]–[27], however, have limitations in that the upper bound of the settling time relies on initial system states or multiple control parameters. This dependency greatly diminishes their practical application in control systems. The prescribed-time control (PTC) introduced in [28] has garnered a lot of attention due to its capability of arbitrarily presetting the settling time, which remains unaffected by the initial states and control parameters. The result gave rise to the proposal of numerous efficacious PTC techniques [29]–[34]. Although the finite/fixed/prescribed-time control methods mentioned above can ensure that the tracking error converges to a neighborhood centered on zero within a finite time, the range of the convergence domain depends on multiple design parameters, i.e., the tracking accuracy cannot be accurately predicted. To this end, some control methods combining PPC and finite/fixed/prescribed-time stabilization are proposed [35]–[38]. Obviously, the combination of the two increases the complexity of control design to some extent. Recently, Liu *et al.* [39] proposed a prescribed-time PPC (PTPPC) algorithm by designing an improved PPF, which can guarantee that the tracking error reaches the specified tracking accuracy within a prescribed time. Afterwards, the PTPPC methods were extend to nonlinear systems with different structures [40]–[44], which effectively improve the performance of the control system. However, in both the SPPC and FTPPC methods, the constraint boundary of tracking error shows a "funnel" shape, where the upper and lower performance

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boundaries are positive and negative functions respectively (like subgraph (a) of Fig. 1). The above methods are very conservative in constraining the transient performance of tracking error before system stabilization. Obviously, the imposition of transient performance constraints in the aforementioned works is undoubtedly a challenging yet indispensable task. The transient stage may give rise to undesirable phenomena, such as overshooting and chattering, which can significantly compromise the closed-loop performance or even render it unacceptable in various engineering applications. To this end, Ji *et al.* [45] proposed a tunnel prescribed control approach that can effectively enhance the system's transient performance. Recently, Shi *et al.* [46] presented an enhancing PPC approach using monotone tube boundaries, under which more key indicators (including tracking accuracy, convergence time and overshoot) were considered. However, the methods proposed in [45], [46] are proposed on the premise of ignoring input saturation, in other words, the above method assumes that the control input can be infinite, which is obviously inconsistent with the requirements of the actual control system.

The presence of input saturation, a commonly encountered phenomenon in practical applications, can significantly compromise the tracking capability and even result in system instability [47]–[50]. The issue of input saturation has been extensively addressed thus far, with the main goal being to convert saturated input into a manageable normal input using diverse transformations. To just name a list, two PPC methods of ISNSs were obtained in [51], [52], in which an auxiliary system is designed to handle the input saturation. Combining the adaptive estimation approach and Nussbaum functions, Wang *et al.* [53] presented an adaptive PPC approach for ISNSs with unknown directions. What is noteworthy is that the above methods address performance constraints and input saturation as separate issues, and assume that they can be implemented concurrently. In fact, performance constraints and input saturation always go hand in hand, for example in cruise control systems [54] and spacecraft systems [55], where cruise accuracy and energy savings are required at the same time. While both of them are usually mutually affecting contradictions, i.e., when the input is saturated, replacing the actual control input with a saturation threshold usually makes the tracking error approach or cross the boundary of the performance constraint, causing a singular problem. On the contrary, if the performance constraint is too harsh, it will lead to input saturation. Recently, Yong *et al.* [56] first established the relationship between input saturation and performance constraints, which can realize a trade-off between the two. Afterwards, the idea was extended to MIMO systems [57], [58], time-delay systems [59] and switched systems [60]. However, the above methods can not simultaneously take into account the tracking accuracy, convergence time and overshoot, which are widely concerned in actual production. The above analysis lead us to study a novel FPPC method for ISNSs, which can not only take into account the important performance indicators of the control system (including the steady state accuracy, convergence time and overshoot), but also achieve a good balance between input saturation and performance constraints.

To sum up, the focus of this article is on the FPPC of ISNSs with unmeasurable states, which has yet to be resolved in previous research. Conclude the main contributions as follows

1) Unlike SPPC methods [14]–[18] and PTPPC methods [39]–[44], which either ignore the convergence time or the overshoot. By designing a novel flexible prescribed-time performance function (FPTPF) and a NMF, the FPPC method presented in this paper simultaneously takes into account more key performance indicators that are of great concern in actual industrial production, including the steady state accuracy,convergence time and overshoot. From this point of view, the proposed FPPC algorithm has better control performance than the SPPC method and PTPPC method.
2) Different from the existing PPC methods for ISNSs [51]–[53], they usually deal with input saturation and performance constraints independently, although the methods proposed in [56]–[60] consider the relationship between the two, while they can not simultaneously take into account the steady state accuracy, convergence time and overshoot. The FPPC method presented in this paper not only simultaneously considers the steady state accuracy, convergence time and overshoot, but also establishes the relationship between the performance constraint and input saturation by introducing an auxiliary system, which makes that the proposed FPPC algorithm realizes a trade-off between them, i.e., when the control input exceeds the saturation threshold, the performance boundary will adaptively increase to mitigate the impact of input saturation on the tracking performance, so as to effectively avoid singularity, when the control input is within the saturation threshold range, the performance constraint boundary can adaptively revert back to the original performance boundary.

II. Problem Formulation and Preliminaries

A. Problem Formulation

The considered nonlinear system is shown as follows

$$\begin{cases} \dot{\xi}_j = f_j\left(\bar{\xi}_j\right) + \xi_{j+1}, j = 1, \ldots, n-1 \\ \dot{\xi}_n = f_n\left(\bar{\xi}_n\right) + u\left(v\right) \\ y = \xi_1 \end{cases} \tag{1}$$

where $\bar{\xi}_n = \left[\xi_1, \ldots, \xi_n\right]^{\mathrm{T}}$ denotes system state vector, $\bar{\xi}_j = \left[\xi_1, \ldots, \xi_j\right]^{\mathrm{T}}$, and $\xi_j (j \geq 2)$ is unmeasured. $y, u\left(v\right)$ represent system output and input, respectively, v represents the controller that needs to be designed. $f_j\left(\cdot\right)$ stands for an uncertain continuous nonlinear function with $j = 1, \ldots, n$. The control input is subject to saturation as follows

$$u\left(v\right) = \begin{cases} v, & |v| \leq u_d \\ u_d \mathrm{sgn}(v), & |v| > u_d \end{cases} \tag{2}$$

where u_d represents the known saturation threshold. The following estimation is employed to address the acute angles of $u\left(v\right)$

$$u\left(v\right) = g\left(v\right) + h\left(v\right) \tag{3}$$

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with $g(v) = u_d \tanh(v/u_d), h(v) = u(v) - g(v)$, where $|h(v)| \le u_d(1 - \tanh(1)) = \bar{h}$.

The aim of this article is to develop an output feedback control algorithm that guarantees

1) all closed-loop signals of system (1) are semi-global bounded;

2) the system output can track the desired signal within a prescribed time, and the tracking error always kept within a flexible performance boundaries that depend on input and output constraints

In order to accomplish the control objectives, the following assumptions are proposed

Assumption 1. [10], [11]: The tracking signal y_d and its first-order derivative is bounded and continuous.

Assumption 2: For function $f_j(\cdot)$ in system (1), there exist a set of constants b_j, such that $\forall Z_1, Z_2 \in R^i$, one has

$$|f_j(Z_1) - f_j(Z_2)| \le b_j \|Z_1 - Z_2\|. \quad (4)$$

Remark 1: It is worth noting that Assumptions 1-2 are reasonable and commonly adopted in the control of nonlinear systems [10], [11], [31]. In fact, numerous practical systems do satisfy Assumption 2, like single-link manipulator systems, permanent magnet brush DC motor systems and so on.

B. Flexible Prescribed-Time Performance Function

In this article, the tracking error $e_1 = \xi_1 - y_d$ (y_d denotes the known desired signal) should satisfy the following performance

$$E_l(t) < e_1(t) < E_h(t) \quad (5)$$

where $[E_l(t), E_h(t)]^{\mathrm{T}} = [e_l(t), e_h(t)]^{\mathrm{T}} + \Lambda_1 \tanh(\eta(t))$, $[e_l(t), e_h(t)]^{\mathrm{T}} = \mathrm{sgn}(e_1(0))(\rho(t) - \rho_T)\mathcal{I}_2 + \Lambda_2 \rho(t)$, $\mathcal{I}_2 = [1, 1]^{\mathrm{T}}$, $\Lambda_k = [-\lambda_k, \lambda_k]^{\mathrm{T}}$ with $k = 1, 2$, $\lambda_1 \in R^+, \lambda_2 \in [0, 1]$, and

$$\rho(t) = \begin{cases} \mathrm{csch}(\rho_0 + \frac{\alpha t}{T-t}) + \rho_T, & t \in [0, T) \\ \rho_T, & t \in [T, +\infty) \end{cases} \quad (6)$$

where $\rho_0, \rho_T, T, \alpha \in R^+$ are the design parameters. $\eta(t)$ represents the output of the following auxiliary system

$$\dot{\eta}(t) = -m_1 \eta(t) + m_2(\sigma_1(t) + \sigma_2(t)), \eta(0) = 0 \quad (7)$$

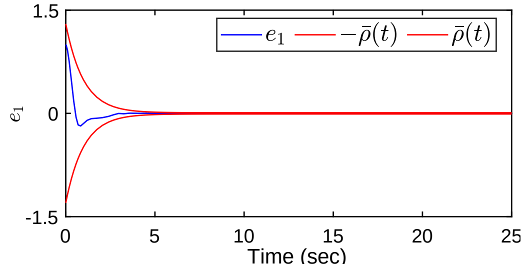
where $\sigma_1(t) = [\mathrm{sgn}(v - u_d) + 1](v - u_d), \sigma_2(t) = [\mathrm{sgn}(v + u_d) - 1](v + u_d)$, $m_1, m_2 \in R^+$.

Remark 2: One can easily know from the expressions of $\sigma_1(t)$ and $\sigma_2(t)$ that $\sigma_1(t) + \sigma_2(t) \ge 0$. Then, one can further know from (7) that if $\eta(0) = 0$, then $\eta(t) \ge 0$ holds for $t \ge 0$. Furthermore, It can be further known from the expressions of $\sigma_1(t)$ and $\sigma_2(t)$ that if and only if $|v| > u_d$, $\sigma_1(t) + \sigma_2(t) > 0$, and when $|v| \le u_d$, $\sigma_1(t) + \sigma_2(t) \equiv 0$, which means that when the control input exceeds the saturation threshold, the performance boundary can adaptively increase to mitigate the impact of input saturation on the tracking performance, so as to effectively avoid singularity, when the control input is within the saturation threshold range, the performance constraint boundary can adaptively revert back to the original performance boundary.

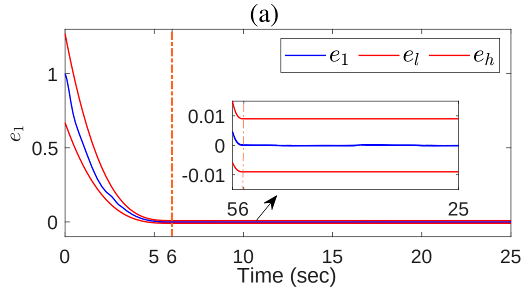
Remark 3: The performance constraint boundary of the proposed FPPC method actually consists of two parts, i.e., $(e_l(t), e_u(t))$ and $(-\lambda_1 \tanh(\eta(t)), \lambda_1 \tanh(\eta(t)))$. The former is to ensure stable and transient performance of the system (including the steady state accuracy, convergence time and overshoot); the latter is designed to maintain controllability when saturation occurs, that is, to ensure system control by sacrificing transient performance. To illustrate this more clearly, we present three different forms of performance functions in Fig. 1.

- In subgraph (a) of Fig. 1, $\bar{\rho}(t)$ represents the PPF of the SPPC approaches [14]–[18], which is generally expressed as $\bar{\rho}(t) = (\bar{\rho}_0 - \bar{\rho}_\infty)e^{-lt} + \bar{\rho}_\infty$ with $\bar{\rho}_0, \bar{\rho}_\infty, l$ be the positive design parameters. Obviously, the SPPC approaches proposed in [14]–[18] can not to constrain the overshoot of tracking errors. Furthermore, the PPF employed in SPPC approaches possesses a characteristic whereby the tracking error can only reach the predetermined convergence region as time tends towards infinity. In other words, the tracking error cannot reach the predetermined steady state accuracy within a prescribed time.
- In subgraph (b) of Fig. 1, the PPF is the special form of the proposed FPTPF, i.e., $\lambda_1 = 0$. Compare with the SPPC approach, both the steady state accuracy (i.e., $\lambda_2 \rho_T$), convergence time (i.e.,T) and overshoot (i.e.,$\lambda_2 \rho_T / e_1(0)$) are considered, which means that the control performance of the proposed FPPC method is superior to that of SPPC methods [14]–[18] and PTPPC methods [39]–[44].
- In subgraph (c) of Fig. 1, the performance function is the proposed FPTPF. On the one hand, it has the performance of taking into account the steady state accuracy, convergence time and overshoot. On the other hand, the proposed FPPC approach considers the internal relationship between performance constraint and input saturation. In fact, when input saturation occurs, the tracking performance is bound to weaken. In this case, if the original performance function is used, the tracking error will contact or even exceed the performance constraint boundary, resulting in the system losing control. The designed FPTPF, however, has the capability to automatically adjust based on the relationship between the control input and the saturation threshold, i.e., when the control input exceeds the saturation threshold, the performance boundary will adaptively expansion according to the difference between the control input and the saturation threshold to mitigate the impact of input saturation on the tracking performance, so as to effectively avoid singularity, when the control input is within the saturation threshold range, the performance constraint boundary can adaptively revert back to the original performance boundary.

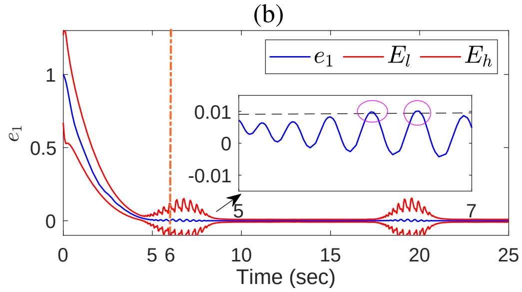
Remark 4: It is noted that the PPC proposed in [56]–[60] also considered the influence between input constraints and performance constraints. While the PPF designed in [56]–[60] shows a “funnel” shape, where the upper and lower performance boundaries are positive and negative functions respectively, which means that the approaches proposed in



(a)



(b)



(c)

Fig. 1: Simulation results under three different PPFs.

[56]–[60] can not to constrain the overshoot of tracking errors and the convergence time. However, the proposed method can adjust the performance function adaptively according to the initial error, and simultaneously take into account multiple key indicators (including the steady state accuracy, convergence time and overshoot), which are widely demanded in industrial production.

C. Fuzzy Approximation

Lemma 1 [33]: The FLS can be employed for the purpose of approximating an uncertain nonlinear function $\mathcal{F}(\mathcal{Z})$, i.e.,

$$\mathcal{F}(\mathcal{Z}) = \Upsilon^{\mathrm{T}}\Omega(\mathcal{Z}) + \phi(\mathcal{Z}), \quad (|\phi(\mathcal{Z})| \leq \phi, \phi \in R^{+}) \tag{8}$$

where $\Omega(\mathcal{Z}) = [\Omega_1(\mathcal{Z}), \ldots, \Omega_m(\mathcal{Z})]^{\mathrm{T}} / \sum_{j=1}^{m} \Omega_j(\mathcal{Z})$, $m \in R^{+}$, Υ, $\Omega(\mathcal{Z})$, $\phi(\mathcal{Z})$, $\mathcal{Z}$ denote weight, basis function, error and input, respectively. $\Omega_i(\mathcal{Z})$ is selected as

$$\Omega_j(\mathcal{Z}) = \exp\left[\frac{-(\mathcal{Z} - \vartheta_j)^{\mathrm{T}}(\mathcal{Z} - \vartheta_j)}{{\iota_j}^2}\right], j = 1, \ldots, m \tag{9}$$

where ϑ_j and ι_j denote the spreads and the center vector of $\Omega_j(\mathcal{Z})$.

III. Main Results

A. State Observer Design

In this article, the following state observer is introduced to approximate unmeasurable states

$$\begin{cases} \dot{\hat{\xi}}_j = \hat{\xi}_{j+1} + \varsigma_j\left(\xi_1 - \hat{\xi}_1\right), j = 1, \ldots, n-1 \\ \dot{\hat{\xi}}_n = u(v) + \varsigma_n\left(\xi_1 - \hat{\xi}_1\right) \end{cases} \tag{10}$$

where $\varsigma_j > 0$ $(j = 1, \ldots, n)$ is suitably selected to ensure that the polynomial $\alpha^n + \sum_{j=1}^{n} \varsigma_j \alpha^{n-j}$ is Hurwitz. Thus, for a given matrix $R = R^{\mathrm{T}} > 0$, there exists $Q = Q^{\mathrm{T}}$ satisfying

$$A^{\mathrm{T}}Q + QA = -R \tag{11}$$

with

$$A = \begin{bmatrix} -\varsigma_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\varsigma_{n-1} & 0 & \cdots & 1 \\ -\varsigma_n & 0 & \cdots & 0 \end{bmatrix}.$$

Let $\tilde{\xi}_j = \xi_j - \hat{\xi}_j$, $B = [0, \ldots, 0, 1]^{\mathrm{T}} \in R^n$, from (1) and (10), one has

$$\dot{\tilde{\xi}} = A\tilde{\xi} + Bu(v) + \bar{f} + \Delta f \tag{12}$$

where $\bar{f} = [\bar{f}_1, \ldots, \bar{f}_n]^{\mathrm{T}}$ with $\bar{f}_j = f_j(\bar{\hat{\xi}}_j)$ and $\bar{\hat{\xi}}_j = \left[\hat{\xi}_1, \ldots, \hat{\xi}_n\right]^{\mathrm{T}}$, $\Delta f = [\Delta f_1, \ldots, \Delta f_n]^{\mathrm{T}}$ with $\Delta f_j = f_j(\bar{\xi}_j) - f_j(\bar{\hat{\xi}}_j)$, $\tilde{\xi} = \left[\tilde{\xi}_1, \ldots, \tilde{\xi}_n\right]^{\mathrm{T}}$.

Choose the following Lyapunov function (LF)

$$V_0 = \tilde{\xi}^{\mathrm{T}}Q\tilde{\xi}. \tag{13}$$

Then, one has

$$\dot{V}_0 \leq -\tilde{\xi}^{\mathrm{T}}R\tilde{\xi} + 2\tilde{\xi}^{\mathrm{T}}Q\bar{f} + 2\tilde{\xi}^{\mathrm{T}}Q\Delta f. \tag{14}$$

According to the mean inequality, one has

$$2\tilde{\xi}^{\mathrm{T}}Q\bar{f} \leq \|\tilde{\xi}\|^2 + \|Q\|^2 L\left(\bar{\hat{\xi}}_n\right) \tag{15}$$

where $L\left(\bar{\hat{\xi}}_n\right) = \sum_{j=1}^{n} f_j^2(\bar{\hat{\xi}}_j)$.

The FLS is used to approximate the continuous nonlinear function $\|Q\|^2 L(\bar{\hat{\xi}}_n)$ as follows

$$\|Q\|^2 L\left(\bar{\hat{\xi}}_n\right) = \Upsilon^{\mathrm{T}}\Omega\left(\bar{\hat{\xi}}_n\right) + \phi\left(\bar{\hat{\xi}}_n\right) \tag{16}$$

where $\Omega(\bar{\hat{\xi}}_n)$ represents the bounded basis function satisfying $\|\Omega(\bar{\hat{\xi}}_n)\| \leq 1$, Υ denotes the unknown weight vector, and $\phi(\bar{\hat{\xi}}_n)$ denotes the approximation error satisfying $\phi(\bar{\hat{\xi}}_n) \leq \phi_0$, ϕ_0 is unknown constant.

Based on Assumption 2, one has

$$\begin{aligned} 2\tilde{\xi}^{\mathrm{T}}Q\Delta f &\leq 2\|\tilde{\xi}\|\|Q\|\left(|\Delta f_1| + \cdots + |\Delta f_n|\right) \\ &\leq 2\|\tilde{\xi}\|\|Q\| \sum_{i=1}^{n}\left(b_i \sum_{j=1}^{i} |\tilde{\xi}_j|\right) \\ &\leq \Theta\|\tilde{\xi}\|\left(|\tilde{\xi}_1| + \cdots + |\tilde{\xi}_n|\right) \\ &\leq \Theta\|\tilde{\xi}\|^2 \end{aligned} \tag{17}$$

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where $\Theta = 2\|Q\|\sum_{j=i}^{n} b_j$.

Let $c_0 = \lambda_{\min R}/\lambda_{\max Q} - (1+\Theta)/\lambda_{\min Q} > 0$, where $\lambda_{\min *}$ and $\lambda_{\max *}$ denote, respectively, the minimum and maximum eigenvalue of $*$. Then, one can further obtain

$$\dot{V}_0 \le -\lambda_{\min R}\|\tilde{\xi}\|^2 + (1+\Theta)\|\tilde{\xi}\|^2 + \delta_0 \tag{18}$$

where $\delta_0 = \|\Upsilon\| + \phi_0$.

B. Prescribed-Time Output Feedback Controller Design

Design a NMF as follows

$$\zeta(t) = \ln\left(\frac{s(t)}{1-s(t)}\right) \tag{19}$$

where

$$s(t) = \frac{e_1(t) - E_l(t)}{E_h(t) - E_l(t)}$$

According to the expression of $s(t)$, it can be obtained that if $e_1(0) \in (E_l(0), E_h(0))$, then $s(0) \in (0,1)$. According to (19), one can obtain that $\zeta(t) \to \infty$ only and only if $s(t) \to 1^-$ or $s(t) \to 0^+$. It means that as long as $s(0) \in (0,1)$ and $\zeta(t)$ is bounded, then $s(t) \in (0,1)$ holds for $\forall t \ge 0$.

Since $e_1(0) \in (E_l(0), E_h(0))$ means $s(0) \in (0,1)$, and $s(t) \in (0,1)$ means $e_1(t) \in (E_l(t), E_h(t))$. Then, one can further obtain that as long as $e_1(0) \in (E_l(0), E_h(0))$ and $\zeta(t)$ is bounded, then one has

$$E_l(t) < e_1(t) < E_h(t). \tag{20}$$

In the following sections, for simplicity's sake, let $E_l(t) = E_l, E_h(t) = E_h$, $\zeta(t) = \zeta, s(t) = s$.

Define the error variables as follows

$$\begin{cases} z_1 = \zeta \\ z_j = \hat{\xi}_j - \alpha_{j-1}, j = 2,\dots,n-1 \\ z_n = \hat{\xi}_n - \alpha_{n-1} - o, \\ \tilde{\Psi}_j = \Psi_j - \hat{\Psi}_j, j = 1,\dots,n \end{cases} \tag{21}$$

where α_j denotes the virtual control function, o denotes an auxiliary signal, which will be given later. $\hat{\Psi}_j$ denotes the estimation of Ψ_j, $\Psi_j = \|\Upsilon_j\|^2$.

Step 1: Choose the first LF candidate (LFc) V_1 as follows

$$V_1 = V_0 + \frac{1}{2}z_1^2 + \frac{1}{2r_1}\tilde{\Psi}_1^2 \tag{22}$$

where $r_1 \in R^+$ denotes a design constant.

According to (19) and (21), one has

$$\dot{z}_1 = \mu\dot{e}_1 + \Gamma \tag{23}$$

where

$$\mu = \frac{1}{s(1-s)(E_h - E_l)}$$

$$\Gamma = \frac{\mu\left[\dot{E}_l(e_1 - E_h) - \dot{E}_h(e_1 - E_l)\right]}{E_h - E_l}.$$

Combining (1), (10) with (23), one can obtain

$$\dot{z}_1 = \mu\left(f_1(\xi_1) + \tilde{\xi}_2 + z_2 + \alpha_1 - \dot{y}_d + \frac{\Gamma}{\mu}\right). \tag{24}$$

Based on (22) and (24), one has

$$\dot{V}_1 = \dot{V}_0 + \mu z_1\left(\bar{\mathcal{F}}_1 + \tilde{\xi}_2 + \alpha_1\right) - \frac{\tilde{\Psi}_1\dot{\hat{\Psi}}_1}{r_1} \tag{25}$$

where $\bar{\mathcal{F}}_1 = f_1(\xi_1) + z_2 - \dot{y}_d + \Gamma/\mu$.

On account of Young's inequality, one has

$$\mu z_1\tilde{\xi}_2 \le \frac{\mu^2 z_1^2}{4} + \|\tilde{\xi}\|^2 \tag{26}$$

Substituting (18) and (26) to (25), one can get

$$\begin{aligned}\dot{V}_1 \le &-(\lambda_{\min R} - 2 - \Theta)\|\tilde{\xi}\|^2 + \delta_0 \\ &+ \mu z_1(\mathcal{F}_1(\mathcal{Z}_1) + \alpha_1) - \mu^2 z_1^2 - \frac{\tilde{\Psi}_1\dot{\hat{\Psi}}_1}{r_1}\end{aligned} \tag{27}$$

where $\mathcal{Z}_1 = [\xi_1, \hat{\xi}_2, y_d]^{\mathrm{T}}$, $\mathcal{F}_1(\mathcal{Z}_1) = \bar{\mathcal{F}}_1 + (5\mu z_1)/4$.

On the basis of Lemma 1, we use a FLS to approximate $\mathcal{F}_j(\mathcal{Z}_j)$, i.e., $\mathcal{F}_j(\mathcal{Z}_j) = \Upsilon_j^{\mathrm{T}}\Omega_j(\mathcal{Z}_j) + \phi_j(\mathcal{Z}_j)$, where $|\phi_j(\mathcal{Z}_j)| \le \phi_j$, and $\phi_j \in R^+$.

In the light of the mean inequality and $0 < \Omega_j^{\mathrm{T}}(\cdot)\Omega_j(\cdot) \le 1$, one can derive

$$\mu z_1\mathcal{F}_1(\mathcal{Z}_1) \le \frac{\mu^2 z_1^2\Psi_1}{2a_1\|\Omega_1(\mathbb{Z}_1)\|^2} + \mu^2 z_1^2 + \bar{\delta}_1 \tag{28}$$

where $\bar{\delta}_1 = a_1/2 + \phi_1^2/4$, $a_1 \in R^+$,$\mathbb{Z}_1 = [\xi_1, y_d]^{\mathrm{T}}$.

Design α_1 and $\dot{\hat{\Psi}}_1$ as follows

$$\alpha_1 = -\frac{c_1 z_1}{\mu} - \frac{\mu z_1\hat{\Psi}_1}{2a_1\|\Omega_1(\mathbb{Z}_1)\|^2} \tag{29}$$

$$\dot{\hat{\Psi}}_1 = \frac{r_1\mu^2 z_1^2}{2a_1\|\Omega_1(\mathbb{Z}_1)\|^2} - \sigma_1\hat{\Psi}_1 \tag{30}$$

where $c_1, \sigma_1 \in R^+$ denote the design constants.

Substituting (28)-(30) to (27), one can further derive

$$\begin{aligned}\dot{V}_1 \le &-(\lambda_{\min R} - 2 - \Theta)\|\tilde{\xi}\|^2 - c_1 z_1^2 \\ &+ \frac{\sigma_1}{r_1}\tilde{\Psi}_1\hat{\Psi}_1 + \delta_1\end{aligned} \tag{31}$$

where $\delta_1 = \bar{\delta}_1 + \delta_0$.

Step j $(j = 2,\dots,n-1)$: Choose the jth LFc V_j as

$$V_j = \frac{1}{2}z_j^2 + \frac{1}{2r_j}\tilde{\Psi}_j^2 + V_{j-1} \tag{32}$$

where $r_j \in R^+$denotes a design constant.

Then, one has

$$\dot{V}_j = z_j\left(z_{j+1} - \dot{\alpha}_{j-1} + \varsigma_j\tilde{\xi}_1 + \alpha_j\right) - \frac{\tilde{\Psi}_j\dot{\hat{\Psi}}_j}{r_j} + \dot{V}_{j-1}. \tag{33}$$

In accordance with Young's inequality, we can get

$$\varsigma_j z_j\tilde{\xi}_1 \le \frac{\varsigma_j^2 z_j^2}{4} + \|\tilde{\xi}\|^2. \tag{34}$$

Substituting (34) to (33), one can obtain

$$\dot{V}_j \le \dot{V}_{j-1} + \|\tilde{\xi}\|^2 + z_j(\mathcal{F}_j(\mathcal{Z}_j) + \alpha_j) - z_j^2 - \frac{\tilde{\Psi}_j\dot{\hat{\Psi}}_j}{r_j} \tag{35}$$

where $\mathcal{F}_j(\mathcal{Z}_j) = z_{j+1} - \dot{\alpha}_{j-1} + (\varsigma_j^2 z_j)/4 + z_j$, $\mathcal{Z}_j = [\xi_1, \hat{\xi}_2, \dots, \hat{\xi}_{j+1}, y_d]^{\mathrm{T}}$.

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Similar to (28), one can derive

$$z_j \mathcal{F}_j(\mathcal{Z}_j) \leq \frac{z_j^2 \Psi_j}{2a_j \|\Omega_j(\mathbb{Z}_j)\|^2} + z_j^2 + \bar{\delta}_j \tag{36}$$

where $\bar{\delta}_j = a_j/2 + \phi_j^2/4$, $\mathbb{Z}_j = [\xi_1, \hat{\xi}_2, \ldots, \hat{\xi}_j, y_d]^{\mathrm{T}}$,$a_j \in R^+$.

Design α_j and $\dot{\hat{\Psi}}_j$ as follows

$$\alpha_j = -c_j z_j - \frac{z_j \hat{\Psi}_j}{2a_j \|\Omega_j(\mathbb{Z}_j)\|^2} \tag{37}$$

$$\dot{\hat{\Psi}}_j = \frac{r_j z_j^2}{2a_j \|\Omega_j(\mathbb{Z}_j)\|^2} - \sigma_j \hat{\Psi}_j \tag{38}$$

where $c_j, \sigma_j \in R^+$ denote the design parameters.

Substituting (36)-(38) into (35), we can further derive

$$\dot{V}_j \leq -(\lambda_{\min R} - j - 1 - \Theta)\|\tilde{\xi}\|^2 + \delta_j - \sum_{m=1}^{j} c_m z_m^2 + \sum_{m=1}^{j} \frac{\sigma_m}{r_m} \tilde{\Psi}_m \hat{\Psi}_m \tag{39}$$

where $\delta_j = \delta_0 + \sum_{m=1}^{j} \bar{\delta}_m$.

Step n: The introduction of an auxiliary system is proposed to counteract the impact of input saturation.

$$\dot{o} = -o + (g(v) - v). \tag{40}$$

The nth LFc V_n is selected as

$$V_n = \frac{1}{2} z_n^2 + \frac{1}{2r_n} \tilde{\Psi}_n^2 + V_{n-1} \tag{41}$$

where $r_n \in R^+$ denotes a design constant.

Then, we have

$$\dot{V}_n = z_n \left(h(v) + v - \dot{\alpha}_{n-1} + \varsigma_n \tilde{\xi}_1 + o \right) - \frac{\tilde{\Psi}_n \dot{\hat{\Psi}}_n}{r_n} + \dot{V}_{n-1}. \tag{42}$$

In accordance with Young's inequality, we can derive

$$z_n h(v) \leq \frac{z_n^2}{4} + \bar{h}^2 \tag{43}$$

$$z_n \varsigma_n \tilde{\xi}_1 \leq \frac{\varsigma_n^2 z_n^2}{4} + \|\tilde{\xi}\|^2. \tag{44}$$

Substituting (43)-(44) to (42), one has

$$\dot{V}_n \leq \dot{V}_{n-1} + \|\tilde{\xi}\|^2 + z_n(v + \mathcal{F}_n(\mathcal{Z}_n) + o) + \bar{h}^2 - z_n^2 - \frac{\tilde{\Psi}_n \dot{\hat{\Psi}}_n}{r_n} \tag{45}$$

where $\mathcal{F}_n(\mathcal{Z}_n) = -\dot{\alpha}_{n-1} + (\varsigma_n^2 z_n + 5z_n)/4$, $\mathcal{Z}_n = [\xi_1, \hat{\xi}_2, \ldots, \hat{\xi}_n, y_d]^{\mathrm{T}}$.

Similar to (28) and (36), one has

$$z_n \mathcal{F}_n(\mathcal{Z}_n) \leq \frac{z_n^2 \Psi_n}{2a_n \|\Omega_n(\mathbb{Z}_n)\|^2} + z_n^2 + \bar{\delta}_n \tag{46}$$

where $\bar{\delta}_n = a_n/2 + \phi_n^2/4$, $a_n \in R^+$,$\mathbb{Z}_n = \mathcal{Z}_n$.

Design v and $\dot{\hat{\Psi}}_n$ as follows

$$v = -c_n z_n - o - \frac{z_n \hat{\Psi}_n}{2a_n \|\Omega_n(\mathbb{Z}_n)\|^2} \tag{47}$$

$$\dot{\hat{\Psi}}_n = \frac{r_n z_n^2}{2a_n \|\Omega_n(\mathbb{Z}_n)\|^2} - \sigma_n \hat{\Psi}_n \tag{48}$$

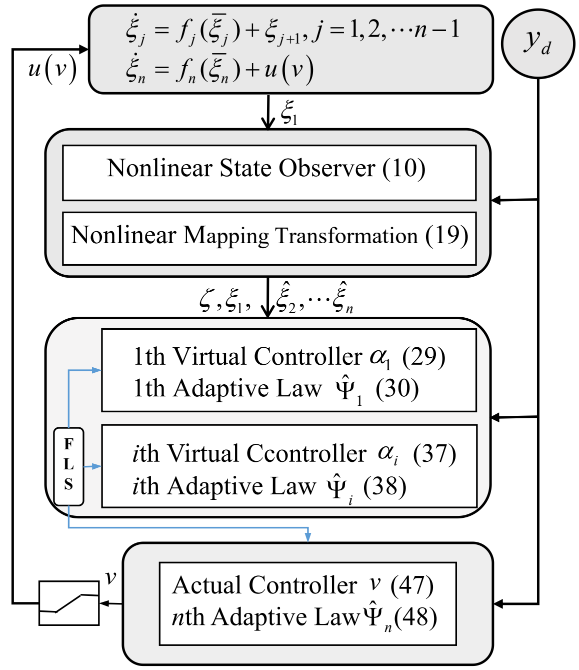

Fig. 2: The block diagram of the proposed method.

where $c_n, \sigma_n \in R^+$ denote the design parameters..

Substituting (46)-(48) into (45), one can derive

$$\dot{V}_n \leq -(\lambda_{\min R} - n - 1 - \Theta)\|\tilde{\xi}\|^2 + \delta_n - \sum_{m=1}^{n} c_m z_m^2 + \sum_{m=1}^{n} \frac{\sigma_m \tilde{\Psi}_m \hat{\Psi}_m}{r_m} \tag{49}$$

where $\delta_n = \delta_0 + \sum_{m=1}^{n} \bar{\delta}_m + \bar{h}^2$.

Bearing in mind $\tilde{\Psi}_j \hat{\Psi}_j \leq -\tilde{\Psi}_j^2/2 + \Psi_j^2/2$, one can derive

$$\dot{V}_n \leq -(\lambda_{\min R} - n - 1 - \Theta)\|\tilde{\xi}\|^2 + \delta - \sum_{m=1}^{n} c_m z_m^2 - \sum_{m=1}^{n} \frac{\sigma_m \tilde{\Psi}_m^2}{2r_m} \tag{50}$$

where $\delta = \delta_n + \sum_{m=1}^{n} (\sigma_m \Psi_m^2)/(2r_m)$.

Define $c = \min_{1 \leq m \leq n} \{(\lambda_{\min R} - n - 1 - \Theta)/\lambda_{\min Q}, 2c_m, \sigma_m\}$ with $\lambda_{\min R} - n - 1 - \Theta > 0$, then one can obtain

$$\dot{V}_n \leq -cV_n + \delta. \tag{51}$$

Remark 5: Note that the proposed method is an intelligent control method, and its advantage is that the designed controller does not involve the expression of specific system functions (see (29)(37)(47)). In other words, the proposed method is suitable for uncertain nonlinear systems. In the control design, FLS is used to approximate the uncertain nonlinear term. In fact, many practical systems have modeling uncertainties, perturbations and other factors, which often lead to the poor effect of traditional control algorithms because of heavy reliance on system models. The block diagram of the proposed method is given in Fig.2.

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C. Stability Analysis

Theorem: Consider nonlinear systems (1), if Assumptions 1-2 and $e_1(0) \in (E_l(0), E_h(0))$ holds, the control algorithm proposed in this article can guarantee that

1) All closed-loop signals in system (1) are semi-globally bounded.
2) the system output can track the desired signal within a prescribed time, and the tracking error always kept within a flexible performance boundaries that depend on input and output constraints.

Proof: 1) According to (51), we can derive

$$\begin{aligned} V_n &\leq V_n(0)\, e^{-ct} + \frac{\delta}{c} \\ &\leq V_n(0) + \frac{\delta}{c}. \end{aligned} \tag{52}$$

From the definition of V_n and (52), one can know that $\tilde{\xi}_j$,z_j and $\tilde{\Psi}_j$ are bounded. On account of (2),(29)-(30),(37)-(38) and (47)-(48), one can obtain that α_j ,$v, u(v)$ and $\hat{\Psi}_j$ are bounded. Since $z_1 = \zeta, z_j = \hat{\xi}_j - \alpha_{j-1}$, so ζ and $\hat{\xi}_j$ are bounded, and so is ξ_j. Accordingly, all the closed-loop signals in system (1) are semi-globally bounded.

2) In accordance with the definition of V_n and (52), one can further derive

$$|z_1| = |\zeta| \leq \sqrt{2\left(V_n(0) + \frac{\delta}{c}\right)} \tag{53}$$

which means that ζ is bound. Based on the expression of $s(t)$ and $e_1(0) \in (E_l(0), E_h(0))$, one can obtain that $s(0) \in (0,1)$. Combining the boundedness of ζ and (19), one can further obtained that

$$E_l < e_1 < E_h. \tag{54}$$

According to the expression of E_l and E_h, one can know that the system output can track the desired signal with a given accuracy within a prescribed time. Meanwhile, the tracking error always kept within a flexible performance boundaries that depend on input and output constraints. ■

IV. Simulation

Example 1 (Theoretical system): Give a system as follows

$$\begin{cases} \dot{\xi}_1 = f_1(\xi_1) + \xi_2 \\ \dot{\xi}_2 = f_2(\bar{\xi}_2) + u(v) \\ y = \xi_1 \end{cases} \tag{55}$$

where $f_1(\xi_1) = \sin \xi_1^2 \cos \xi_1$, $f_2(\bar{\xi}_2) = \xi_1\left(1 + \xi_2^2\right) \sin \xi_2 - 1.2\xi_1\xi_2$. Select the design parameters as $c_1 = 10, c_2 = 5, a_j = 1, r_j = 50, \varsigma_j = 1$ with $j = 1, 2,$ $u_d = 1.5$, $\rho_0 = 0.7$, $\rho_T = 0.03$, $T = 6$, $\lambda_1 = 1, \lambda_2 = 0.3$, $m_1 = 5, m_2 = 0.5$. Let $y_d = \sin(0.5t)$, and $\hat{\Psi}_2(0) = [3, 2]^{\mathrm{T}}$,$\bar{\xi}_2(0) = [1, 0.3]^{\mathrm{T}}$,$\hat{\bar{\xi}}_2(0) = [1, 0.3]^{\mathrm{T}}$. The subgraphs (a)-(b) in Fig.3 demonstrate that the system output effectively tracks the desired signal within the prescribed time, and the tracking error always kept within a flexible performance boundaries that depend on input and output constraints. The curves of $\xi_2, \hat{\xi}_2$ and $v, u(v)$ are depicted in (c)-(d) of Fig.3, respectively. Fig.4 shows the tracking curve and tracking error curve respectively when $e_1 = 0$ and $e_1 < 0$.

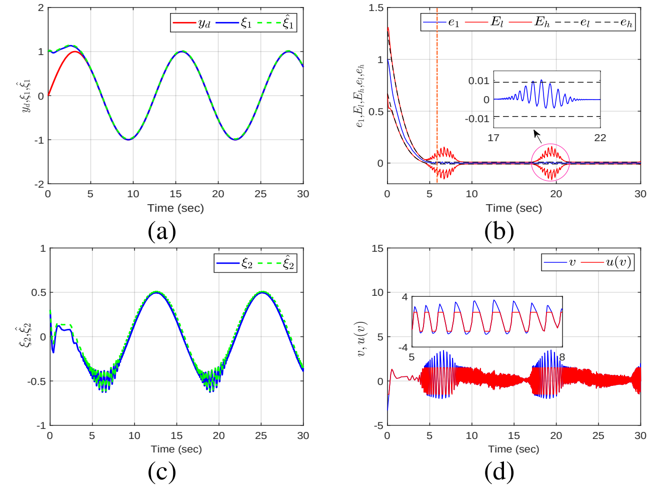

Fig. 3: (a)Curves of $y_d, \xi_1, \hat{\xi}_1$. (b)Curves of e_1, e_l, e_h, E_l, E_h. (c)Curves of $\xi_2, \hat{\xi}_2$. (d)Curves of $v, u(v)$.

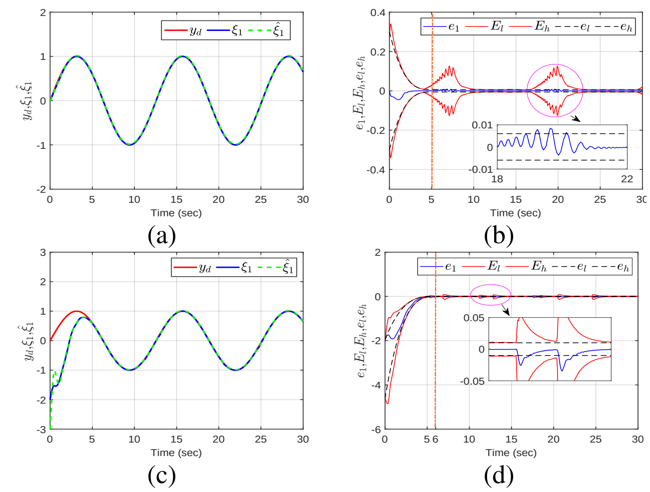

Fig. 4: (a)(b) represents the tracking curve and the tracking error curve when $e_1 = 0$, respectively;(c)(d) represents the tracking curve and the tracking error curve when $e_1 < 0$, respectively.

The proposed FPPC method is compared with the SPPC method [14] in order to further validate its superiority. Let $\bar{\rho}(t) = (\bar{\rho}_0 - \bar{\rho}_\infty)e^{-lt} + \bar{\rho}_\infty$ with $\bar{\rho}_0 = 1.3, l = 1, \bar{\rho}_\infty = 0.009$. Subgraphs (a) and (b) of Fig. 5 respectively show the tracking error curves of the proposed FPPC method and the SPPC method without considering input saturation. It can be seen that the tracking errors of both methods can be contained within the performance boundaries. However, the proposed F-PPC method takes into account the convergence time, tracking accuracy and overshoot, while the SPPC can only ensure the tracking accuracy. Subgraphs (c) and (d) of Fig. 5 respectively show the tracking error curves of the proposed FPPC method and the SPPC method with considering input saturation. Obviously, when input saturation occurs, the tracking error under the SPPC method will touch the constraint boundary and the simulation can not continue, but the proposed FPPC method is still valid.

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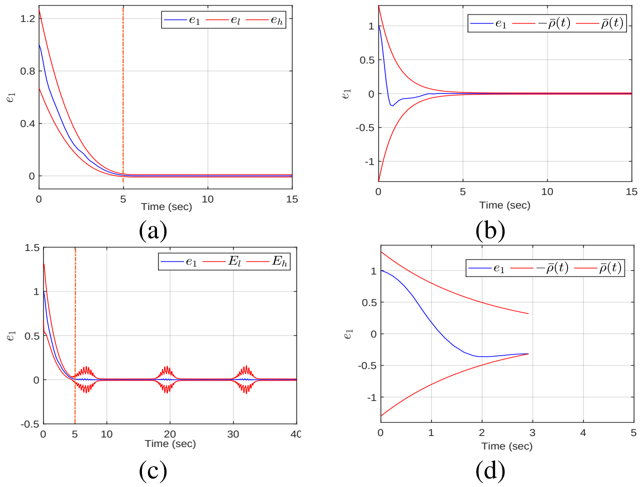

Fig. 5: (a)(b) respectively represent the tracking error curves of the proposed FPPC method and the SPPC in [14] without considering input saturation;(c)(d) respectively represent the tracking error curves of the proposed FPPC method and the SPPC in [14] with considering input saturation.

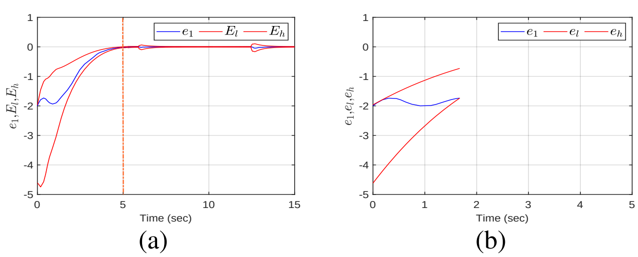

Fig. 6: (a)(b) respectively represent the tracking error curves of the proposed FPPC method and the enhancing PPC in [46].

We also compare the proposed FPPC method with the enhancing PPC method proposed in [46]. In fact, a significant advantage of the proposed FPPC method compared with the enhancing PPC method proposed in [46] is that it takes into account the inevitable input saturation problem in real systems, and the proposed FPPC method reasonably establishes an organic relationship between input saturation and performance constraints. From Fig.6, one can see that when the input is saturated, the singularity will appear under the enhancing PPC method proposed in [46], resulting in the termination of simulation, while the proposed FPPC method is still valid.

Remark 6: The proposed algorithm incorporates several control parameters. As indicated by the stability analysis, the selection of these parameters in practice adheres to the following rules: the tracking error can be reduced by decreasing δ_i or increasing c_i and r_i. One advantage of this paper is the introduction of flexible performance functions, so the tracking accuracy and convergence time can be adjusted by adjusting ρ_T and T according to actual needs. The parameters, in practice, should be integrated with the system structure and actual performance requirements for comprehensive debugging.

Example 2 (Practical system): Consider a single-link robot arm system [10] modeled with the following dynamic equation

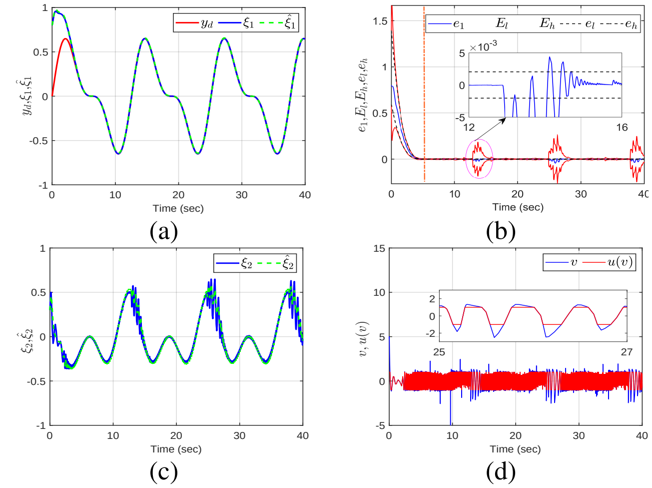

Fig. 7: (a)Curves of $y_d, \xi_1, \hat{\xi}_1$. (b)Curves of e_1, e_l, e_h, E_l, E_h. (c)Curves of $\xi_2, \hat{\xi}_2$. (d)Curves of $v, u(v)$.

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = \frac{u(t)}{M} - \frac{mgl\sin\xi_1}{2M} - \frac{B\xi_2}{M} \\ y = \xi_1 \end{cases} \tag{56}$$

where $m = 0.02kg, g = 9.8m/s^2, l = 1m, M = 1kg, B = 1$. Select the design parameters as $c_1 = 10, c_2 = 5, a_j = 1, r_j = 50, \varsigma_j = 1$ with $j = 1, 2$, $u_d = 1.5$, $\rho_0 = 0.7$, $\rho_T = 0.03$, $T = 6$, $\lambda_1 = 1$,$\lambda_2 = 0.3$, $m_1 = 5, m_2 = 0.5$. Let $y_d = \sin(0.5t)$, and $\hat{\bar{\Psi}}_2(0) = [3, 2]^{\mathrm{T}}$,$\bar{\xi}_2(0) = [1, 0.3]^{\mathrm{T}}$,$\hat{\bar{\xi}}_2(0) = [1, 0.3]^{\mathrm{T}}$. The subgraphs (a)-(b) in Fig.7 demonstrate that the system output effectively tracks the desired signal within the prescribed time, and the tracking error always kept within a flexible performance boundaries that depend on input and output constraints. The curves of $\xi_2, \hat{\xi}_2$ and $v, u(v)$ are depicted in (c)-(d) of Fig.7, respectively.

Similarly, the comparison simulation results between the proposed FPPC method and the SPPC method [14] are shown in Fig.8. Let $\bar{\rho}(t) = (\bar{\rho}_0 - \bar{\rho}_\infty)e^{-lt} + \bar{\rho}_\infty$ with $\bar{\rho}_0 = 1.3, l = 1, \bar{\rho}_\infty = 0.009$. Subgraphs (a) and (b) of Fig. 8 respectively show the tracking error curves of the proposed FPPC method and the SPPC method without considering input saturation. One can see that the tracking errors of both methods can be contained within the performance boundaries. The proposed FPPC method, however, takes into account the convergence time, tracking accuracy and overshoot, while the SPPC can only ensure the tracking accuracy. Subgraphs (c) and (d) of Fig. 8 respectively show the tracking error curves of the proposed FPPC method and the SPPC method with considering input saturation. Obviously, when input saturation occurs, the tracking error under the SPPC method will touch the constraint boundary and the simulation can not continue, but the proposed FPPC method is still valid. Furthermore, we also compare the proposed FPPC method with the enhancing PPC method proposed in [46]. From Fig.9, one can see that when the input is saturated, the singularity will appear under the enhancing PPC method proposed in [46], resulting in the termination of simulation, while the proposed FPPC method

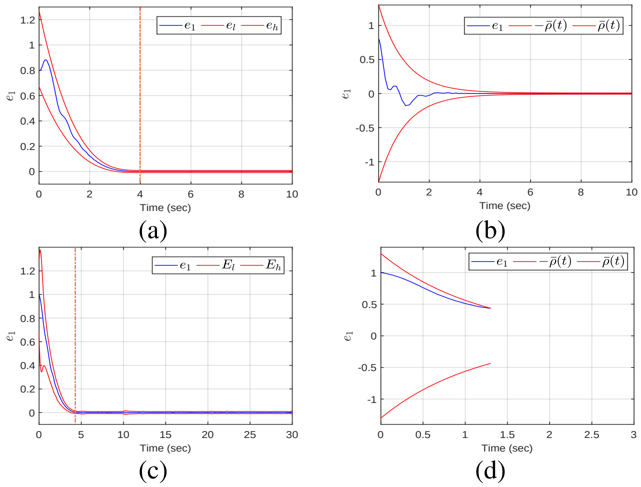

Fig. 8: (a)(b) respectively represent the tracking error curves of the proposed FPPC method and the SPPC in [14] without considering input saturation;(c)(d) respectively represent the tracking error curves of the proposed FPPC method and the SPPC in [14] with considering input saturation.

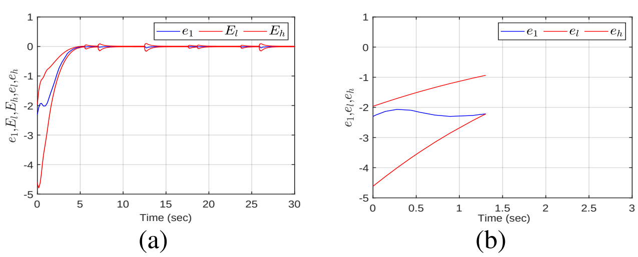

Fig. 9: (a)(b) respectively represent the tracking error curves of the proposed FPPC method and the enhancing PPC in [46].

is still valid.

V. Conclusion

This article first presents a FPPC algorithm for ISNSs with unmeasurable states. By designing the FPTPF that depend on performance constraints and input saturation, the proposed FPPC method not only takes into account multiple key indicators required by industrial production (including the steady state accuracy, convergence time and overshoot), but also achieves a good balance between performance constraints and input saturation. In addition, the state observers are designed to observe the unmeasured states, and the FLSs are applied to approximate the unknown nonlinear functions. The FPPC and application of input saturated markov jump systems [61], [62]and cyber-physical systems [63] will be the direction of our future efforts.

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